SOLUTION: (a) The solar constant $\mathcal{G}_{sc}$ is the value of the energy flux per unit normal area at the distance of the Earth. Using this value we calculate the emissive power of the Sun (called the luminosity L):

$$L = 4\pi d^2 \mathcal{G}_{sc} = 4\pi \cdot \left(1.5 \cdot 10^{11}\,\text{m}\right)^2 \cdot 1370\,\text{W}/\text{m}^2 = 3.9 \cdot 10^{26}\,\text{W}$$

The Sun approximates a black body which does not receive radiation from the surrounding space. Therefore, Equ.(7.36) applies to the relation between emissive power and temperature, leading to a value of

$$T = \left(\frac{L}{4\pi R_{sun}^2 \sigma}\right)^{1/4} = \left(\frac{3.9 \cdot 10^{26}\,\text{W}}{4\pi \cdot \left(7.0 \cdot 10^8\,\text{m}\right)^2 \cdot \sigma}\right)^{1/4} = 5770\,\text{K}$$

for the surface temperature of the Sun.

(b) The entropy flux flowing away from the Sun through the field is given by

$$I_S = \frac{4}{3} A \sigma T_{sun}^3 = 8.95 \cdot 10^{22}\,\text{W}/\text{K}$$

EXAMPLE 7.12. Surface temperature of the Earth.

Model the Earth as (a) a black body of uniform temperature, absorbing radiation from the Sun and emitting radiation to outer space. How large is the value of the temperature attained by the surface of this body in steady state? (b) Repeat this for a gray surface and again (c) for a black radiator that absorbs 70% of sunlight (because of reflection by snow and clouds).

SOLUTION: (a) We can use Equ.(7.67) without convection and T = 0 K for the environment. This yields

$$A_s \mathcal{G}_{sc} = A \sigma T^4$$

A_s is the projected surface of a the sphere (a circle) and A is the surface of the planet. Inserting and solving for T leads to

$$T = \left(\frac{\mathcal{G}_{sc}}{4\sigma}\right)^{1/4} = \left(\frac{1370\,\text{W}/\text{m}^2}{4\sigma}\right)^{1/4} = 279\,\text{K}$$

(b) For non-black bodies we have

$$a_s A_s \mathcal{G}_{sc} = e A \sigma T^4$$

Since $a_s = e$ for gray surfaces, the new condition leads to the same result for the temperature of the planet, T = 279 K.

(c) If the Earth radiates like a black body, e = 1. However, if it reflects 30% of incoming radiation, we may set a_s = 0.7 (it is as if the planet were a selective absorber/radiator). Now we have

$$T = \left(\frac{a_s \mathcal{G}_{sc}}{4 e \sigma}\right)^{1/4} = \left(\frac{0.7 \cdot 1370\,\text{W}/\text{m}^2}{4\sigma}\right)^{1/4} = 255\,\text{K}$$

All three results are too low. The mean surface temperature of our planet is more like 288 K, leaving us with the problem of how to explain this difference (see Chapter 9, Section 9.6).

EXAMPLE 7.13. The radiative heat transfer coefficient.

Write the equation for the exchange of energy between a black body and its surroundings, Equ.(7.38), in a form which resembles the equation of convective heat transfer at a solid–fluid boundary. How would you write the overall heat transfer coefficient, including convection?

SOLUTION: It is possible to transform the term involving the difference of the fourth powers of the temperatures in such a way that the difference of temperatures occurs in the equation:

$$\begin{aligned} I_{E,rad,net} &= \sigma A\left[T_1^4 - T_2^4\right] \\ &= \sigma A\left(T_1^2 + T_2^2\right)\left(T_1^2 - T_2^2\right) \\ &= \sigma A\left(T_1^2 + T_2^2\right)\left(T_1 + T_2\right)\left(T_1 - T_2\right) \end{aligned}$$

Comparison with the desired form, Equ.(7.27), shows that

$$h_{rad} = \sigma\left(T_1^2 + T_2^2\right)\left(T_1 + T_2\right)$$

Obviously, the radiative heat transfer coefficient strongly depends upon the temperatures involved.

If convection is present as well, we are dealing with a case of parallel flow of heat. The flux of energy is equal to the sum of the radiative and the convective fluxes. Therefore, the overall heat transfer coefficient must be equal to the sum of the radiative and convective transfer coefficients.

EXAMPLE 7.14. Absorption of solar radiation: the balance of entropy.

A body absorbs a fraction f of the energy current associated with solar radiation I_E intercepted by it. Represent the losses to the environment in terms of a total heat transfer coefficient h (which includes radiation). Assume that solar radiation does not carry any entropy. (Because of the high temperature associated with solar radiation, this assumption is quite applicable here.) (a) Calculate the sum of the rates of entropy generation due to absorption of radiation and losses. (b) Show that you obtain the same result using the balance of entropy for the body if you take the system boundary to coincide with the environment at temperature T_a. (c) Compare the magnitude of the effects for a body with a surface area of 1.0 m^2 at a temperature of 50°C absorbing 80% of an energy flux of 1000 W/m^2 in an environment of 20°C. The heat transfer coefficient has a value of 10 W/(K·m^2).

SOLUTION: For the solution of the problem we will need the equation of balance of energy for the body:

$$\dot{E} = \Sigma_E - I_{E,loss} = f I_E - hA\left(T - T_a\right)$$

(a) Entropy production is due to two distinct irreversible processes, the absorption of radiation and heat transfer to a colder body (the environment). Since the energy of solar radiation absorbed is dissipated, the rate of production due to absorption of radiation is

$$\Pi_{S1} = \Sigma_E / T$$

The rate of production of entropy as a result of heat transfer, on the other hand, is given by

$$\Pi_{S2} = hA\left(T - T_a\right)\left(\frac{1}{T_a} - \frac{1}{T}\right)$$

(b) If we consider the body as our system and draw the system boundary at the location of the environment at temperature T_a, we include the part responsible for heat transfer in the system. In this case, the equation of balance of entropy takes the form

$$\dot{S} = I_S + \Pi_S$$

Remember that the radiation is assumed not to deliver any entropy, so there is no source term. Now we have

$$\Pi_S = \frac{\dot{E}}{T} - \frac{I_{E,loss}}{T_a} = \frac{f I_E - hA(T - T_a)}{T} + \frac{hA(T - T_a)}{T_a}$$

This result is equivalent to what we obtained by calculating the rates of production independently.

(c) Inserting the numbers into the expression obtained in (a) gives values of 2.5 W/K and 0.095 W/K, respectively. This tells us something about the relative irreversibilities of the processes (absorption and heat loss): the former is much larger. If we wanted to optimize a system by minimizing entropy production, we have to be able to quantify different contributions to irreversibility (see Chapter 9 for a discussion of this approach).

EXERCISES AND PROBLEMS

1. Sunlight passes in one direction through a gas inside a long cylinder. The flux of entropy at the surface where the light is entering has a magnitude of 5.0 W/K. At the opposite end, the flux of the current of entropy leaving the body is 4.0 W/K. (a) Determine the net flux of entropy with respect to the region of space occupied by the body. (b) At what (minimal) rate is the entropy of the body changing? (c) What is the value of the source rate of entropy for the field? How large is the flux of entropy with respect to the material body?
2. A copper bar of length 0.50 m and cross section 10.0 cm^2 has a temperature of 500 K at one end and 300 K at the other. As heat flows through the bar in steady state, measurements indicate that the temperature varies linearly along the bar. (a) Determine the temperature gradient. Take the direction of entropy flow to be positive. (b) Estimate the current densities of entropy and of energy for the center of the bar using the values read from Fig. 7.8. How large is the conductivity with respect to energy? (c) Divide the bar into two equal parts. With this current of entropy flowing, what is the flux of entropy at the surface where the parts touch with respect to the part from where the entropy is flowing?
3. An immersion heater in a water kettle is hooked up to 220 V. Its electrical resistance is 160 Ω at a temperature of 20°C; the temperature coefficient of the resistance is $4 \cdot 100^{-3}$ K^{-1}. If the heat transfer coefficient between heater and water is 100 W/(K·m^2) and the surface area of the heater is 0.020 m^2, how large will the energy current from the heater to the water be? How does the situation change if a layer of mineral deposit builds up around the heater?
4. Show that the energy current transmitted through a cylindrical shell of length L having inner and outer radii r_1 and r_2 is

$$I_E = \pi L \left[\frac{1}{2 r_1 h_1} + \frac{1}{2 r_2 h_2} + \frac{1}{2 k_E} \ln\left(\frac{r_2}{r_1}\right) \right]^{-1} \left(T_1 - T_2\right)$$

   where h_1 and h_2 are the inner and the outer convective heat transfer coefficients. The temperatures of the fluids on the inside and the outside are T_1 and T_2.
5. A cylindrical volume of rock below ground has been heated uniformly to 50°C while the rest of the rock has a temperature of 10°C. (This might be done in solar seasonal heat storage applications.) Use the average values for granite for the properties of the rock. (a) Make

the following model for heat loss from the cylindrical area to the surroundings. While the temperatures of the storage area and the surroundings remain uniform, heat flows through a cylindrical mantle with inner and outer radii equal to half and to twice the radius of the storage cylinder, respectively. Estimate the energy current due to heat loss for a radius of 5.0 m and a length of the cylindrical space of 40 m. (b) How large should the radius be made for heat loss over a period of half a year not to exceed one quarter of the energy stored in the cylinder?

6. A sheet of metal with a selective surface of 2.0 m^2 lies horizontally on the ground. The bottom of the sheet is well insulated. In the visible part of the spectrum the emission coefficient of the metal is 0.90, while in the infrared it is 0.30. Take the ambient temperature to be 20°C. The Sun stands 50° above the horizon, and 70% of the radiation outside the atmosphere penetrates the air. (Assume all the radiation from the sky to be direct and not diffuse.) (a) Neglecting convection, how large should the temperature of the metal sheet be in the light of the Sun? (b) Now take into consideration convective heat transfer at the upper surface of the sheet. The convective heat transfer coefficient is assumed to be 14 W/(K·m^2). Calculate the temperature attained by the sheet under these conditions.

7. Hot water is left to cool in a thin-walled aluminum can. In a first experiment, the aluminum is highly polished (curve number 1 in Fig. P.7). In the second experiment (curve 2), the can is painted black. Assume the convective transfer from the water to the can to be highly efficient. Data: Mass of water: 0.476 kg; surface area of the can: 0.0325 m^2; Ambient temperature: 21.6°C. (a) Determine the rate of change of entropy of the water in Experiment 1 at $t = 500$ s? (b) Assume radiation to be negligible in Experiment 1. What are the convective entropy and energy transfer coefficients for the surface of the can (can to air). (c) Use the data of Experiment 2 to determine the emissivity of the black surface.

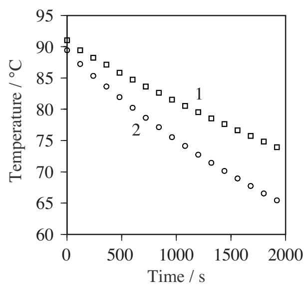

Figure P.7

8. Normally, the surface temperatures of stars are derived from their colors or their spectra. However, it is also possible to calculate this quantity from the intensity of their light (i.e., from the irradiance at the surface of the Earth), and from their angular diameter as seen from the Earth. Angular diameters of some nearby stars can be determined with the aid of interferometric methods. In the case of the star Sirius in the constellation of Canis Majoris, these values are $8.6 \cdot 10^{-8}$ W/m^2, and $6.12 \cdot 10^{-3}$ arc seconds, respectively.

9. Consider the Earth as a uniform body. (a) How large is the rate at which entropy appears in the atmosphere, biosphere, and the oceans of the Earth if we take their temperature to be 300 K? The solar constant outside the atmosphere is 1.36 kW/m^2. 30% of the radiation is directly reflected back into space. (b) How large is the flux of entropy through the radiation field just before radiation is absorbed? (c) How large is the rate of production of entropy on the planet as a result of absorption? (d) How large is the rate of entropy generation overall?

10. A photovoltaic panel with an area of 1.0 m^2 is exposed to constant solar radiation having an energy current of 800 W/m^2. Initially, panel and cells are at ambient temperature (300 K). The panel has a heat capacity (energy capacity) of 1000 J/K. The absorption coefficient of the panel for sunlight is 0.85. The emission coefficient of the panel for thermal radiation is 1. Energy goes directly to the air as well (the heat transfer coefficient is 12 W/(K·m^2)). The electric efficiency of the panel decreases with temperature according to

$$\eta = 0.15 - b\left(T - T_a\right) \quad , \quad b = 1.667 \cdot 10^{-3}\,\mathrm{K}^{-1}$$

The efficiency is defined as the ratio of electric power and energy current of sunlight (not the absorption rate!). (a) What is the electric power right at the beginning? (b) Formulate the law of balance of energy of the panel in general (instantaneous) form. (c) What is the rate of change of temperature of the panel right at the beginning? (d) Determine the steady-state temperature of the panel resulting after a period of time. (e) Sketch as precisely as possible, the temperature and the electric power of the panel as functions of time.

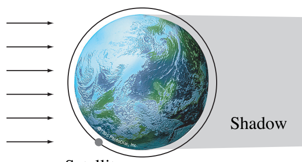

Figure P.11.1

11. A spherical satellite with a radius of 0.50 m moves in a low orbit around the Earth (Fig. P.11.1). Approximately half the time it is exposed to the Sun's light (the solar constant is

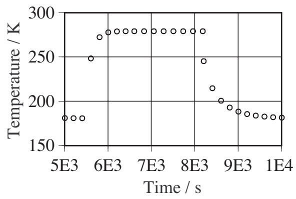

Figure P.11.2

1370 W/m^2). In the Earth's shadow it is irradiated by the earth itself. The satellite is a thin aluminum shell. (a) The Earth absorbs approximately 70% of the energy of the incident light of the Sun. The energy is then uniformly reradiated over the entire surface. What is the energy flow of the earth's radiation per square meter? (b) Calculate the highest and the lowest steady-state temperatures reached by the satellite. This temperature is uniform over the entire surface. Assume that the satellite is a black body radiator. When it is in sunlight, ignore the Earth's radiation. (c) Determine the mass of the satellite with the help of the temperature as a function of time (see Fig. P.11.2).

12. In solar energy applications, parabolic troughs are used to focus light upon absorbers of cylindrical shape. Calculate the heat loss coefficient of such an absorber. Consider it to be made of a metal pipe having a diameter of 5.5 cm, surrounded by a thin glass cover with an outer diameter of 8.5 cm. The annulus between the pipe and the cover is evacuated. Take the convective heat transfer coefficient at the surface of the cover to be 35 W/(K·m^2). The emissivities of glass and the metal pipe are 0.88 and 0.92, respectively. Present the result as a function of absorber temperature for an ambient temperature of 20°C.

13. A bottle of white wine is placed in a refrigerator whose inner temperature we take to be constant at 0°C. How long will it take for the temperature of the wine to decrease from an initial value of 20°C to the desired 8°C? Treat the wine as a uniform system of mass 0.75 kg and use the constitutive quantities of water. The bottle is made out of glass with a thickness of 5.0 mm. The height and the diameter of the main body of the bottle are 25 cm and 8 cm, respectively; neglect its bottom and its neck and treat the mantle as a flat layer. The convective transfer coefficients inside and outside are 200 W/(K·m^2) and 10 W/(K·m^2), respectively.

14. A spherical thin-walled water tank has a volume of 1.0 m^3. The water inside is kept at a constant temperature of 60°C by heating it with an energy current equal to 1.0 kW. The ambient temperature is 15°C. How long will it take for the water to reach a temperature of 40°C after the heater has been turned off?

15. A body of water having a volume of 1.0 m^3 loses heat to its surroundings. The temperatures are 80°C and 20°C for the water (initially) and the environment, respectively. The product of total heat transfer coefficient and surface area is 60 W/K. (a) How long does it take for the temperature difference between the water and the surroundings to decrease to half its initial value? (b) How large is the rate of production of entropy right at the beginning? (c) How much entropy is produced in total from the beginning until the water has cooled down completely? (d) How much energy could have been released by an ideal Carnot engine operating between the water and the environment as the water cools to ambient temperature?

16. To maintain an inner temperature of 20°C in a building situated in a 0°C environment, the required heating load is 5 kW. Without heating, the house is found to cool down as follows: every day, its temperature decreases by 1/5 of the temperature difference to the environment. (a) Determine the product of surface area and total heat transfer coefficient. (b) Model the building as a single node system. Calculate its temperature coefficient of energy. (c) Assume the temperature inside the building to be 12°C. Calculate the heating power necessary if you wish the temperature to rise by 1°C per hour.

17. A tall, well insulated cylinder of radius 0.75 m contains 10,000 kg of water. The lower 3500 kg has a temperature of 20°C, while the temperature of the rest of the water is 80°C. Such stratification may be approximately attained while charging a hot water storage tank in solar applications. (a) Estimate how long it will take for the difference of the temperatures of the two segments of water to decrease to 30°C. (*Hint:* Model the segments as uniform bodies; for the thermal resistance take a distance from the center of the hotter to the center of the cooler part.) (b) Calculate the initial rate of production of entropy. (c) How large is the initial rate of loss of available power?