SOLUTION: In addition to the definition of relative humidity in terms of the vapor pressure and the saturation vapor pressure, we will need to simultaneously solve Equ.(15.21), Equ.(15.33) and Equ.(15.34). For the stated problem, namely finding the wet bulb temperature, the equations are nonlinear. We will have to find the values of enthalpies and pressures for the quantities appearing in the equations, partly for the still unknown value T_{wb}. Below you will find the relations to be set up if the thermodynamic property data are given in the form of programmed functions. Compare the solution of these equations with a solution attempted using tabulated values.

```
Given data:   phi = 0.75
              T1 = 30
              P1 = 101.3
Properties:   ha_T = Enthalpy(Air,T=T1)
              ha_Twb = Enthalpy(Air,T=Twb')
              hg_T = Enthalpy(Steam,T=T1,X=1),
              hg_Twb = Enthalpy(Steam,T=Twb,X=1)
              hl_Twb = Enthalpy(Steam,T=Twb,X=0)
              Pg_Twb = Pressure(Steam,T=Twb,X=1)
              Pg_T = Pressure(Steam,T=T1,X=1)
Relations:    w = ((ha_Twb–ha_T)+w'*(hg_Twb–hl_Twb))/(hg_T–hl_Twb)
              w' = 0.622*Pg_Twb/(P1–Pg_Twb)
              w = 0.622*Pv_T/(P1–Pv_T)
              phi = Pv_T/Pg_T
Solution:     ha_T = 303.6 kJ/kg        ha_Twb = 299.9
              hg_T = 2555               hg_Twb = 2549
              hl_Twb = 110.2
              P1 = 101.3 kPa            Pg_T = 4.246
              Pg_Twb = 3.425            Pv_T = 3.184
              phi = 0.750
              T1 = 30.0°C               Twb = 26.3
              w = 0.0202                w' = 0.0218
```

The solution of the problem was performed completely within the program EES (Klein et al., 1991). The form of the property functions is pretty much self explanatory. *Air* stands for dry air, while *Steam* denotes water (liquid and vapor). The temperature and its wet bulb counterpart are denoted by T and Twb, respectively, while w is used for the humidity ratio. X is the quality (defined in Equ.(15.59); X=0 is for saturated liquid, while X=1 is for saturated vapor).

15.3 EVAPORATION

The term evaporation is most commonly used to describe vaporization of some water from a larger body of water and subsequent transfer of the vapor into the air surrounding the body of water. In this section, evaporation or condensation of water, and the mixing of water vapor with air will be considered from a dynamical perspective. Such processes are important in nature (think of the evaporation of water from lakes, from soil, or from leaves), in chemical engineering (where we let other substances evaporate as well), in power engineering (cooling towers), and in air conditioning.

15.3.1 Evaporation and Diffusion of Water Vapor in Air

Consider a body of water with unmoving air above it. If the air is not saturated, there is a difference of chemical potentials between the liquid water and the water vapor in the air leading to the transfer of more water vapor into the air. Water must evaporate, so the concentration of vapor will be higher in a layer adjacent to the body of water than further away from it. As a result, the vapor diffuses upward into the drier air (see Fig. 15.16).

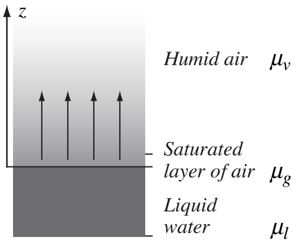

Figure 15.16: Evaporation from a body of water can be understood as the diffusion of water vapor from a saturated layer of air near at the surface of the water to points further away where the air is not saturated.

This process continues until the air above the water is saturated. If it is continuously removed—which normally is the case in an open space—evaporation from the surface of the body of water continues as long as liquid water remains. Evaporation does not depend upon the water being warmer than the air. In fact, the process described works even when the water is cooler than the air. All that is needed is a higher concentration of water vapor near the surface of the water compared to points further away.

Evaporation of warm water: a simple model. Hot water is poured into a glass and placed on a scale. The water stands at a depth d from the top of the container (see Fig. 15.17, left). Mass, water and air temperature, and relative humidity are measured as functions time. The temperature decreases as a result of cooling (Fig. 15.17, right) which is mostly due to evaporation: the mass of the water decreases quite steeply at higher temperatures whereas the flow of vapor into the air decreases to a lower and almost constant value for water temperatures close to the temperature of the air (see Fig. 15.17, center).

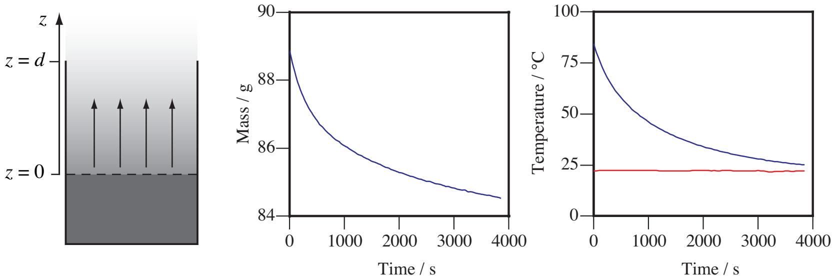

Figure 15.17: Evaporation of water in a glass open at the top (left). Mass (center) and water and air temperature (right) have been measured as functions of time. The relative humidity during the experiment was 0.20.

If evaporation is due to the diffusion of vapor from the surface of the liquid at $z = 0$ a distance d through the air, we can formulate a simple relation for the flow of amount of substance of water vapor (remember Section 6.7):

$$I_{n,v} = -A h_v \left(\bar{c}_d - \bar{c}_0 \right) \qquad \textbf{(15.35)}$$

Here, $\bar{c}$ stands for the concentration of vapor in the air at $z = d$ and $z = 0$, respectively. h_v is transfer coefficient for amount of substance, and A symbolizes the cross section of the water surface. If we treat the vapor as an ideal gas, its concentration is proportional to its partial pressure. Therefore, Equ.(15.35) transforms into

$$I_{n,v} = -\frac{A h_v}{R T_a} \left(\phi P_g \left(T_a \right) - P_g \left(T_w \right) \right) \qquad \textbf{(15.36)}$$

P_g is the saturation vapor pressure and ϕ denotes the relative humidity. Here, I have

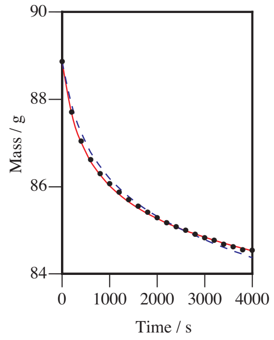

Figure 15.18: Evaporation of hot water from an open container as in Fig. 15.17. Dots are measured values of the mass of the remaining water. The dashed line is the result of the model presented in Equ.(15.36). The solid line results from a version of the model in Equ.(15.44).

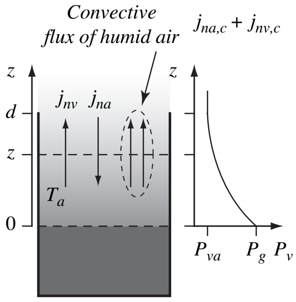

Figure 15.19: Evaporation of water in a container as in Fig. 15.17. There are several different currents of vapor and air (left). The diagram on the right shows the form of the vapor pressure in the column of air.

assumed that the vapor is always at the temperature of the air, T_a, even near the surface of the water which is at T_w. In the layer at the surface of the body of water, the air is saturated. The transfer coefficient h_v will depend upon the depth of the water in the container, and upon the diffusivity of water vapor in air.

If the measured temperatures (Fig. 15.17) are used to predict the flow of mass $I_{m,v} = M_{0v} I_{m,v}$ from the surface of the water, we get the dashed line shown in Fig. 15.18; at first sight, it compares relatively well with the measured mass (dots) but clearly shows different behavior. For this reason. let us extend the model to one which includes the spatial variation of conditions in the container of Fig. 15.17.

Diffusion of vapor in air. First, we have to understand the precise nature of the transports in the container of Fig. 15.17 in the column of moist air above the body of water. Let us assume that there is variability in the vertical direction only, and that the situation is in steady-state. There must certainly be a conductive (diffusive) flux of water vapor whose current density of amount of substance at a point z in the column of humid air will be abbreviated by j_{nv} (Fig. 15.19). In a mixture of gases such as dry air and water vapor, if one of the components diffuses in one direction, there must be an equal diffusive flux of the other component in the opposite direction. This is so because, in steady-state, one mole (or one molecule) of vapor must be replaced by one mole (or one molecule) of air. So we have a current density of dry air j_{na} at point z.

The downward flux of air has important consequences. If there were nothing to balance it, air would accumulate at the surface of the water. Since this is not the case, we need a convective (or bulk) flux of humid air going up where the flux of (dry) air must equal the diffusive downward flux of (dry) air. Since the air is humid, water vapor is transported convectively, so we have an additional flux of water vapor going upward (see Fig. 15.19).

The laws of balance of amounts of substance (of air and of water vapor) and coupling between the fluxes can be formulated quite simply. First, we should note that the fluxes are independent of position z since we have steady-state conditions. Second, the diffusive flux density of air is equal in magnitude to the diffusive flux density of water vapor:

$$j_{nv}(z) = -j_{na}(z) \tag{15.37}$$

Since air has to remain balanced in the column above the body of water, we have

$$j_{na,c}(z) = -j_{na}(z) \tag{15.38}$$

The subscript c stands for convective transports. Finally, there is the convective transport of vapor which is coupled to the convective flux of air. Since the amount of vapor in the air equals the molar fraction of the amount of humid air, we have

$$j_{nv,c}(z) = \bar{x}_v \left(j_{na,c}(z) + j_{nv,c}(z) \right) \tag{15.39}$$

(The current of humid air equals the sum of currents of air and vapor.) Now, the total current density of vapor is the sum of its diffusive and convective parts which leads to:

$$j_{nv,tot}(z) = \frac{1}{1 - \bar{x}_v} j_{nv}(z) \tag{15.40}$$

Now, constitutive relations have to be introduced. For one, the diffusive current density of vapor can be formulated as in Section 6.7.2:

$$j_{nv}(z) = -D\frac{\partial \bar{c}_v}{\partial z} \tag{15.41}$$

(The diffusivity of water vapor in air is about $2.4 \cdot 10^{-5}$ m^2/s at 300 K.) Furthermore, the concentration of water vapor can be expressed in terms of the pressure of this component by using the ideal gas relation:

$$\bar{c}_v(z) = \frac{1}{RT_a}P_v(z) \tag{15.42}$$

For isothermal situations, the temperature is equal to the ambient temperature everywhere. If we introduce Equations (15.41) and (15.42) into Equ.(15.40) and integrate the first order differential equation from $z = 0$ to $z = d$, and remember that the current densities are constant, we obtain

$$I_{nv} = D\frac{P}{RT_a}\frac{A}{d}\ln\left(\frac{P - P_{va}}{P - P_g}\right) \tag{15.43}$$

A denotes the cross section of the container from which water evaporates, and P_{va} and P_g are the water vapor pressure in the air far from the container (given by its temperature and relative humidity) and the saturation vapor pressure at the same temperature, respectively.

Evaporation of hot water. Let us revisit the experiment described in Fig. 15.17. The water is hotter than the environment for most of the time, so we do not have isothermal conditions as assumed in the derivation performed above. If we assume the column of air above the water to be at T_a, and the water at T_w, we could apply Equ.(15.43) and get a simple result:

$$I_{nv} = D\frac{P}{RT_a}\frac{A}{d}\ln\left(\frac{P - P_{va}(T_a)}{P - P_g(T_w)}\right) \tag{15.44}$$

Interestingly, the pressure dependence of this expression fits data well, as shown by the solid line and the dots in Fig. 15.18. However, for an actual fit, the current in Equ.(15.44) has to be multiplied by a constant factor of the order of 10. While the form of Equ.(15.44) seems to apply to the evaporation of hot water from a container, the strength of the current is considerably larger than predicted by the simple model. Furthermore, experiments with different values of the depth of the water level do not show the dependence on d expected from Equ.(15.43) or (15.44). Vapor condenses on the inside of the glass container and evaporates again at different heights and joins the vapor coming from the surface of the body of water. This may explain the differences.

15.3.2 Evaporative Cooling

Evaporative cooling can be taken to mean two different things. On the one hand, bodies (solids, liquids) are cooled if a liquid at their surfaces evaporates. We have seen this effect several times already such as in the drop of temperature of a wet thermometer (Fig. 6.8). On the other hand, we may direct our attention to the air that is involved in evaporation and takes up vapor; its temperature is reduced. This plays a role in air-conditioning: in a hot and dry climate, we may want the comfort of cooler and more humid air. Below, two applications will be discussed: forcing dry air through a wet pad and the cooling of already cold water sitting in a container.

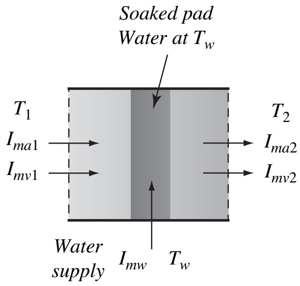

Figure 15.20: Relatively dry, hot air flows through a pad soaked with water. Because of evaporation, the temperature of the humid air exiting the duct is lowered, and its humidity is increased.

Dry air flowing through a wet pad. Consider a simple duct through which air can flow. The temperature of the air is to be lowered by evaporation of water in a pad through which the air is forced (Fig. 15.20). If we assume the system to be thermally insulated and if we can neglect mechanical and gravitational effects, the situation is similar to the one discussed in adiabatic saturation (Section 15.2.3) or to the case of mixing of streams of fluids (Section 8.6.4). For this reason, only the laws of balance will be formulated here.

There are three inflows and two outflows (Fig. 15.20). The steady-state balances of mass for air and water take the form

$$\begin{aligned} I_{ma1} &= I_{ma2} \\ I_{mv1} + I_{mw} &= I_{mv2} \end{aligned} \tag{15.45}$$

The law of balance of entropy contains six terms: one each for the incoming and outgoing substances and the production rate:

$$s_{a1} I_{ma1} + s_{v1} I_{mv1} + s_w I_{mw} + \Pi_S = s_{a2} I_{ma2} + s_{v2} I_{mv2} \tag{15.46}$$

The law of balance of energy only involves the convective currents of energy for the five currents in Fig. 15.20:

$$h_{a1} I_{ma1} + h_{v1} I_{mv1} + h_w I_{mw} = h_{a2} I_{ma2} + h_{v2} I_{mv2} \tag{15.47}$$

At both he inlet and the outlet, the currents of mass of vapor and air are related by the humidity ratios defined in Equ.(15.20). To complete a model, we need expressions for the entropies and enthalpies of air and water or steam for the conditions of temperature, pressure, and humidities that apply in the situation depicted in Fig. 15.20.

Evaporation of cold water. Water having the same temperature as the environment can still become colder by evaporation if it is inside a well insulated container that is open at the top. Historically, in dry climates, water was kept cool by storing it in amphoras which were sprayed with water, and fans were used to keep the air flowing over the wet containers.

Here is an experiment and dynamical model that shows how to deal with this situation. Water having roughly the temperature of the environment is poured into a relatively well insulated container that is open at the top. It is placed on an automatic scale, and water and air temperature, and relative humidity of the environment, are measured as functions of time (Fig. 15.21). We need to track the entropy and mass of the water and the entropy of the container in our model (it turns out that the dynamics of the container has to be taken into account if we want to get a good fit between model and data; see Fig. 15.21, center):

$$\begin{aligned} \dot{S}_w &= -I_{S,evap} - I_{S,cond} \\ \dot{m}_w &= -I_{m,evap} \end{aligned} \tag{15.48}$$

$$\dot{S}_c = -I_{S,ca} + I_{S,cond} \tag{15.49}$$

Note that, in principle, there should be a conductive entropy current going directly from the water to the air (or vice-versa); however, the simulation is not affected greatly by this addition.

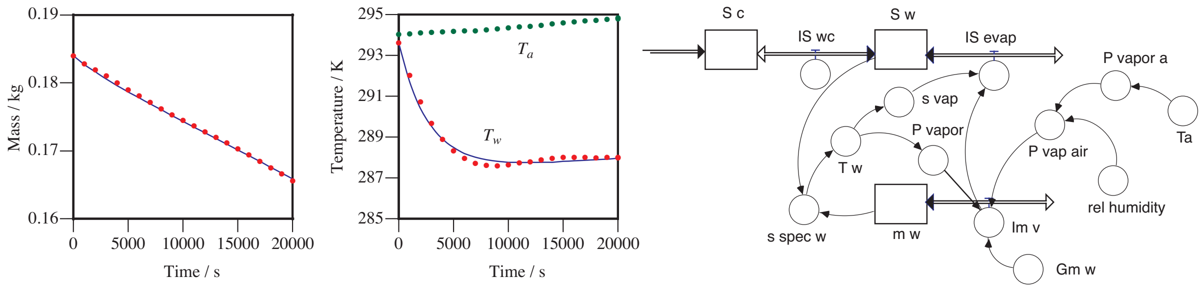

Figure 15.21: Evaporation of cold water in a relatively well insulated container that is open at the top. Mass (left) and water and air temperature (center) have been measured as functions of time (dots denote data). The relative humidity during the experiment was 0.15. Right: Diagram of a system dynamics model. Note that there is more to the model not shown here (top left of the diagram). Here, *w*, v, *a*, and *c* denote water, vapor, air, and container, respectively.

Constitutive laws have been assumed to be as follows. Since temperature differences are fairly small, entropy production due to conductive transfer has been neglected and transport models have been assumed to be linear. In particular, the flow of water evaporating from the surface of the water in the tank is modeled as in Equ.(15.36):

$$I_{m,v} = -G_{m,v}\left(\phi P_g\left(T_a\right) - P_g\left(T_w\right)\right) \tag{15.50}$$

The relation has been formulated for the flux of mass rather than for amount of substance. $G_{m,v}$ is a conductance for mass transfer. Conductive currents of entropy have been written accordingly. Note that the temperature of the water in the tank can be calculated on the basis of its specific entropy which equals the ratio of instantaneous values of entropy and mass. Together with expressions for vapor pressure and the specific entropy of vapor, the missing quantities can be calculated. Here is the relation for the evaporative current of entropy:

$$I_{S,evap} = s_{vap} I_{m,v} \tag{15.51}$$

where s_{vap} is the specific entropy of the steam leaving the surface of the body of water. The model performs quite well (see the solid lines in the diagrams on the left and at the center of Fig. 15.21).

15.3.3 Cooling Towers

Consider cooling towers as a last application of evaporative cooling. Where the cooling needed for a large thermal power plant cannot be done by transferring the entropy to a river, a cooling tower may be used. Here, water used for the condenser of the plant is sprayed into a stream of relatively dry air so that a part of the water evaporates. The portion of the cooling water that does not evaporate has given up entropy. Together with makeup water (used to replenish the flow of water through the condenser), the colder cooling water is returned to the condenser of the plant.

The processes going on can be envisioned in a schematic similar to the one showing evaporative cooling in Fig. 15.20. In place of the wet pad, we have cooling water falling through the stream of air forced through the device. (Naturally, the geometry of an actual cooling tower will be different from what we discuss here, but the processes can still be understood by the schematic shown in Fig. 15.22.)

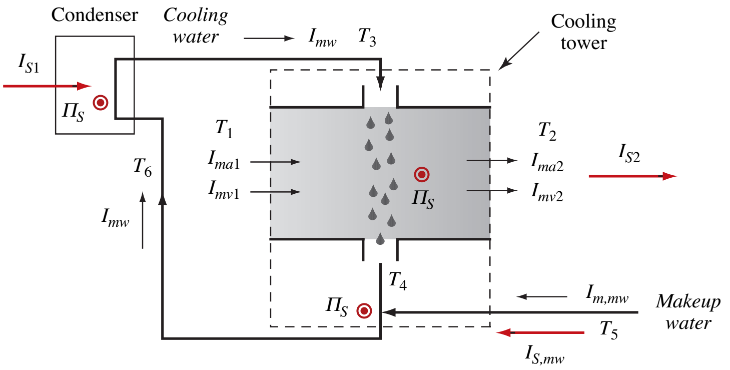

Figure 15.22: Schematic of a cooling tower. The entropy that is rejected by the power plant (I_{S1}) has to be communicated to the environment.

Assume the power plant is designed to eject a current of entropy I_{S1} to the cooling water. At least this amount of entropy will have to be carried away by the moisturized air leaving the cooling tower.

We can calculate the performance of a cooling tower using the type of (steady-state) model discussed above and applied in Example 15.6 and Example 15.7. To exercise a less formal approach, consider the following simplified model that is suitable for estimates. The entropy coming from the power plant (I_{S1}) has to be communicated to the environment (I_{S2}). If we assume the entropy current leaving the cooling tower with the humid air to be equal to the one from the power plant, we neglect entropy production in the tower and the entropy current with makeup water. (Entropy production is mostly due to mixing of water, vapor, and air at different conditions, and the dissipation of the energy used for pumping, which is neglected here anyway.)

If we assume further that the temperature of the air flowing though the cooling tower does not change much, its entropy will not either. Therefore, all the entropy communicated to the tower from the power plant has to be in the water that is evaporated and added to the airflow:

$$I_{S2} = \left(s_{v2}\omega_2 - s_{v1}\omega_1\right)I_{ma} \qquad \textbf{(15.52)}$$

This is so because ωI_{ma} is the current of water vapor in air if I_{ma} is the current of dry air (Equ.(15.20)). s_v denotes the specific entropy of vapor at its temperature and (partial) pressure. If we specify the condition of the air at the inlet and outlet of the cooling tower, we can calculate the humidity ratios according to what we have learned in the previous sections. Calculating the entropies poses a small problem. Equ.(15.52) shows that we cannot just work with the difference of the entropies at two states. To deal with this case properly, we have to make sure we have the entropy with respect to a well defined state that holds for both points 1 and 2. This can be insured by setting the entropy of liquid water equal to zero at $T = 273$ K and $P = 101.3$ kPa. The entropy of the vapor can be calculated from this if the entropy of vaporization and the temperature coefficient of enthalpy of the vapor are known (Example 15.9).

QUESTIONS

9. What kind of reasoning leads to the expression in Equ.(15.35)?
10. Why are the fluxes in the model of Fig. 15.19 spatially constant (independent of z)?
11. Why does the current of vapor in evaporation inside a container depend upon the ratio of the pressures of the dry air far from the water and near its surface?
12. If hot dry air is cooled by evaporating some water (as in Fig. 15.20), what happens to the exit temperature of the stream of air if the flow of water is increased? Do you expect there to be a minimum of entropy production for some water current, or does the entropy production rate vary monotonically?
13. Why is it possible to neglect entropy production in the model shown in Fig. 15.21
14. What are the conditions necessary for the entropy current leaving a cooling tower to be equal to the one rejected by the power plant? Where do you expect these conditions to be violated most strongly?

EXAMPLE 15.7. Cooling air by humidification.

Hot (40°C) dry ($\phi = 0.1$) air flows through an evaporative cooler as in Fig. 15.20. The current of mass of the air alone is 1.0 kg/s. How much water needs to be added each second to cool the air to 28°C? (The water added has a temperature of 25°C) What will the relative humidity of the air at that point be? Ambient pressure is taken to be 1.0 atm.

SOLUTION: We formulate the laws of balance of energy and mass, the constitutive relations, and the properties of the substances in EES (Klein et al., 1991). The laws of balance take the forms

```
ha_T1*I_ma_1 + hg_T1*I_mv_1 + hw_Tw*I_mw_1 =
                    ha_T2*I_ma_2 + hg_T2*I_mv_2
I_ma_2 = I_ma_1
I_mv_2 = I_mv_1 + I_mw_1
```

The constitutive relation needed are

```
Pv_T1 = phi_1*Pg_T1
w_1 = 0.622*Pv_T1/(P1-Pv_T1)
I_mv_1 = w_1*I_ma_1
w_2 = I_mv_2/I_ma_2
Pv_T2 = w_2/(w_2 + 0.622)*P1
phi_2 = Pv_T2/Pg_T2
```

and the pressures and enthalpies can be evaluated with the help of

```
Pg_T1 = Pressure(Steam,T=T1,X=1)      ha_T1 = Enthalpy(Air,T=T1)
hg_T1 = Enthalpy(Steam,T=T1,X=1)      ha_T2 = Enthalpy(Air,T=T2)
hg_T2 = Enthalpy(Steam,T=T2,X=1)      hw_Tw = Enthalpy(Steam,T=Tw,X=0)
```