Optimization in a Simple Model of the Earth's Atmosphere

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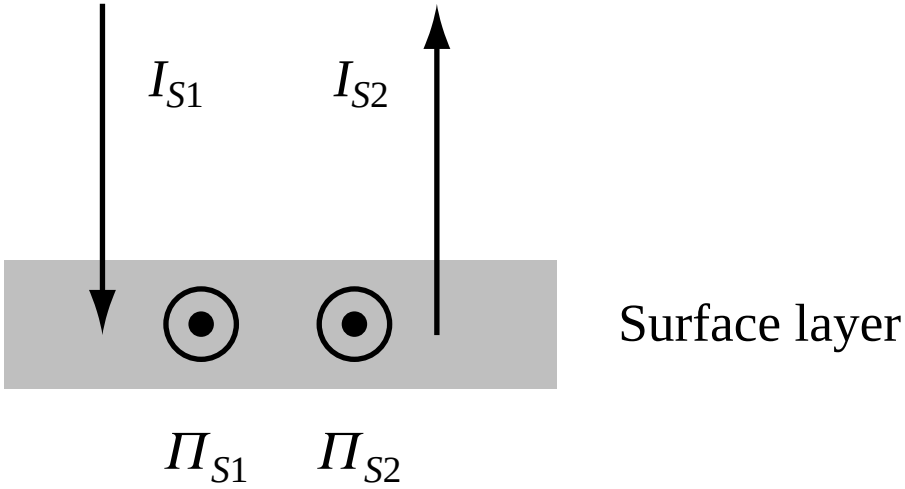
1 TEMPERATURES OF GROUND AND ATMOSPHERE

A. The Planet Without an Atmosphere



Law of balance

$$0 = I_{S1} + I_{S2} + \Pi_{S1} + \Pi_{S2}$$

Entropy currents

$$I_{S1} = \frac{1}{4} A \frac{4}{3} \frac{G_{sc}}{T_s}$$

$$I_{S2} = -A\sigma \frac{4}{3} T_E^3$$

Entropy production rates

$$\Pi_{S1} = \left(\frac{3}{4} \frac{T_s}{T_E} - 1 \right) I_{S1}$$

$$\Pi_{S2} = -\frac{1}{4} I_{S2}$$

Minimal total entropy production rate

$$T_E = \left(\frac{G_{sc}}{4\sigma} \right)^{0.25}$$

Model Using Second Law

Parameters

sigma = 5.67e-8

T_s = 5777

R_s = 7.0e8

d = 1.5e11

albedo = 0.3

a_g = 1

Entropy fluxes

I_s = 1/4*(1-albedo)*4/3*sigma*T_s^3*(R_s/d)^2

I_s_out = a_g*4/3*sigma*T_g^3

Rate of entropy production

Pi_s = 1/4*I_s_out + (3/4*T_s/T_g-1)*I_s

Law of balance of entropy

I_s_out = I_s + Pi_s

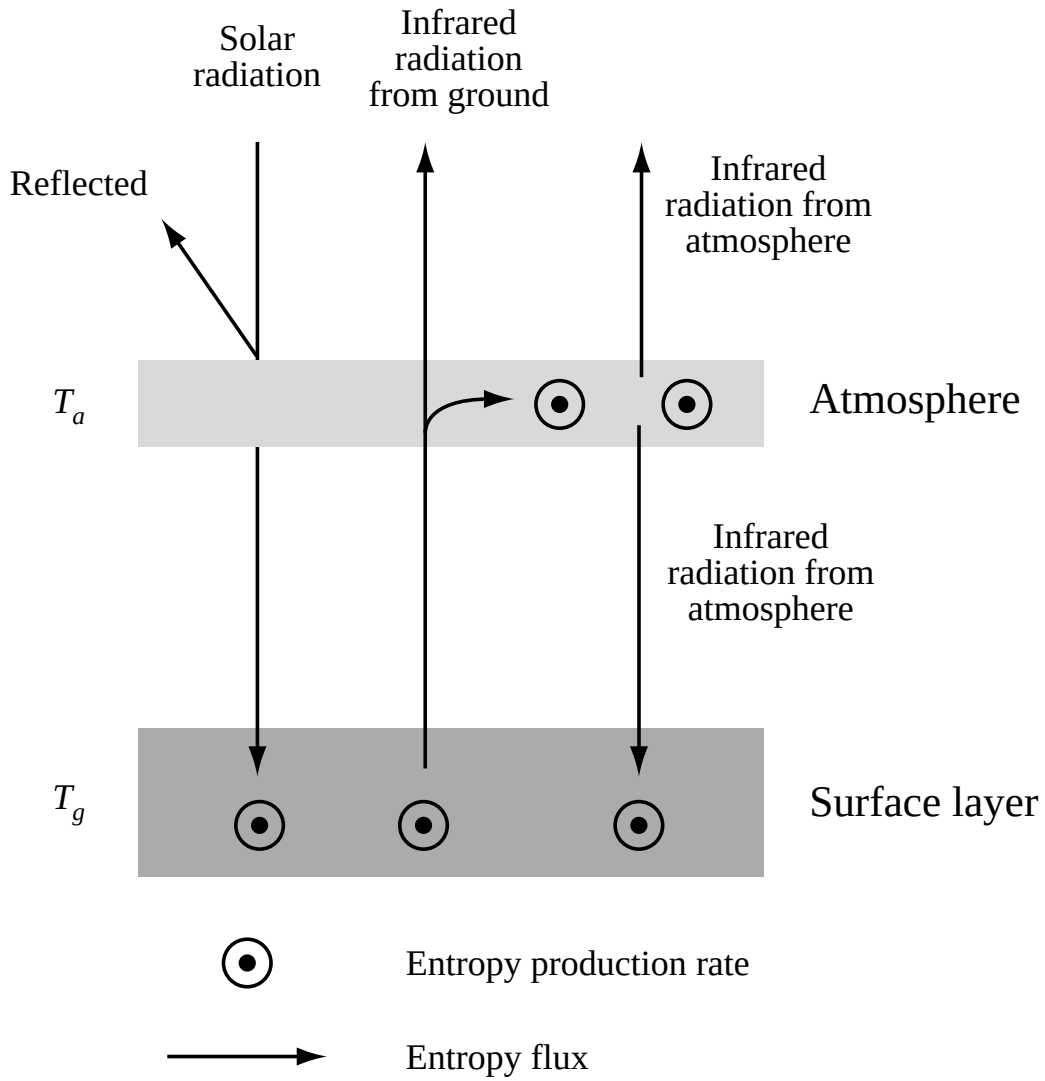
SOLUTION

I_s_out = 1.257

Pi_s = 1.202

T_g = 255.2

B. The Planet With a Single Layer of an Atmosphere



Model Using Second Law

Parameters	sigma = 5.67e-8
	a_a = 0.87
	a_g = 1
	T_s = 5777
	R_s = 7.0e8
	d = 1.5e11
	albedo = 0.30
Constitutive laws for ground	
Rate of absorption	
of entropy from sun:	A_sgs = 1/4*(1-albedo)*4/3*sigma*T_s^3*(R_s/d)^2
Emittance:	E_sg = 4/3*a_g*sigma*T_g^3
Radiosity:	B_sg = E_sg + (1-a_g)*B_sa
Entropy production rate:	Pi_sg = 1/4*E_sg + (3/4*T_a/T_g - 1)*a_g*B_sa + (3/4*T_s/T_g - 1)*A_sgs
Constitutive laws for atmosphere	
Emittance:	E_sa = 4/3*a_a*sigma*T_a^3
Radiosity:	B_sa = E_sa
Entropy production rate:	Pi_sa = 2*1/4*E_sa + (3/4*T_g/T_a - 1)*a_a*B_sg

Laws of balance

Ground: A_sgs + a_g*B_sa + Pi_sg = E_sg

Atmosphere: a_a*B_sg + Pi_sa = 2*E_sa

SOLUTION

A_sgs = 0.05555

B_sa = 0.998

B_sg = 1.929

E_sa = 0.998

E_sg = 1.929

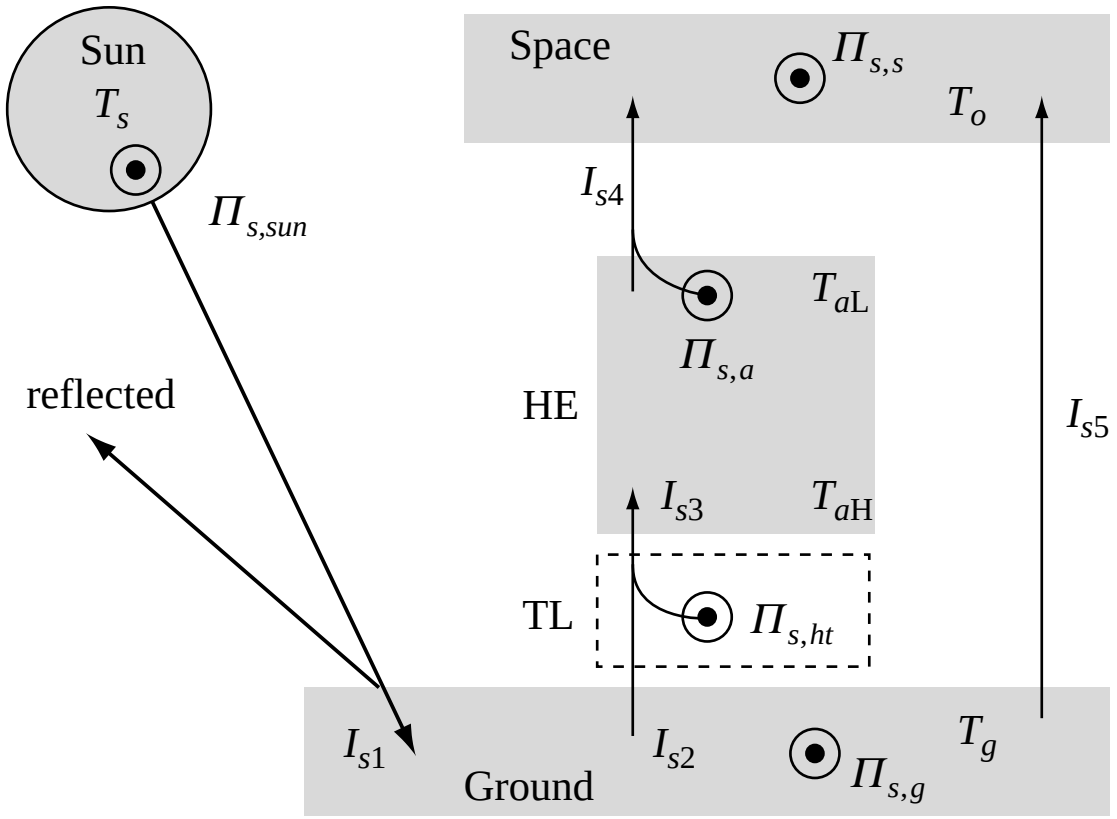
Pi_sa = 0.318

Pi_sg = 8.76e-1

T_a = 247.6

T_g = 294.4

2 THE WINDS AS THE RESULT OF A SOLAR THERMAL ENGINE



Laws of balance

$$I_{S1} + \Pi_{S,g} = I_{S2} + I_{S5}$$

$$I_{S2} + \Pi_{S,ht} = I_{S3}$$

$$I_{S3} + \Pi_{S,a} = I_{S4}$$

Entropy currents

$$I_{S1} = 0.25(1 - albedo)\left(\frac{R_s}{d}\right)^2 \frac{4}{3}\sigma T_s^3$$

$$I_{S2} = \frac{h}{T_g}\left(T_g - T_{a1}\right)$$

$$I_{S4} = \frac{4}{3}\sigma T_{a2}^3$$

$$I_{S5} = \frac{4}{3}\sigma T_g^3$$

Entropy production rates

$$\Pi_{S,g} = \left(\frac{3}{4} \frac{T_s}{T_g} - 1 \right) I_{S1} + \frac{1}{4} I_{S5}$$

$$\Pi_{S,ht} = \frac{1}{T_{a1}} \left(T_g - T_{a1} \right) I_{S2}$$

$$\Pi_{S,a} = \frac{1}{4} I_{S4}$$

$$\Pi_{S,s} = \left(\frac{3}{4} \frac{T_{a2}}{T_o} - 1 \right) I_{S4} + \left(\frac{3}{4} \frac{T_g}{T_o} - 1 \right) I_{S5}$$

Power of the ideal thermal engine

$$\mathcal{P}_{HE} = \left(T_{a1} - T_{a2} \right) I_{S3}$$

Model Using Second Law

Parameters	…
	T_o = 3
	h = 20
	a_ua = 1
Entropy fluxes	
	A_sgs = 1/4*(1-albedo)*4/3*sigma*T_s^3*(R_s/d)^2
	I_s2 = 1/T_g*h*(T_g-T_aH)
	I_s4 = 4/3*a_ua*sigma*T_aL^3
	I_s5 = 4/3*a_g*sigma*T_g^3
Entropy production rates	
Transfer layer:	Pi_sht = (T_g-T_aH)*I_s2/T_aH
Ground:	Pi_sg = (3/4*T_s/T_g - 1)*A_sgs + 1/4*I_s5
Atmosphere:	Pi_satm = 1/4*I_s4
Space:	Pi_sspace = (3/4*T_aL/T_o-1)*I_s4 + (3/4*T_g/T_o-1)*I_s5
Sun:	Pi_ssun = 1/4*4/3*sigma*T_s^3*(R_s/d)^2
Law of balance of entropy	
Ground:	A_sgs + Pi_sg = I_s2 + I_s5
Transfer layer:	I_s3 = I_s2 + Pi_sht
Atmosphere:	I_s4 = I_s3 + Pi_satm

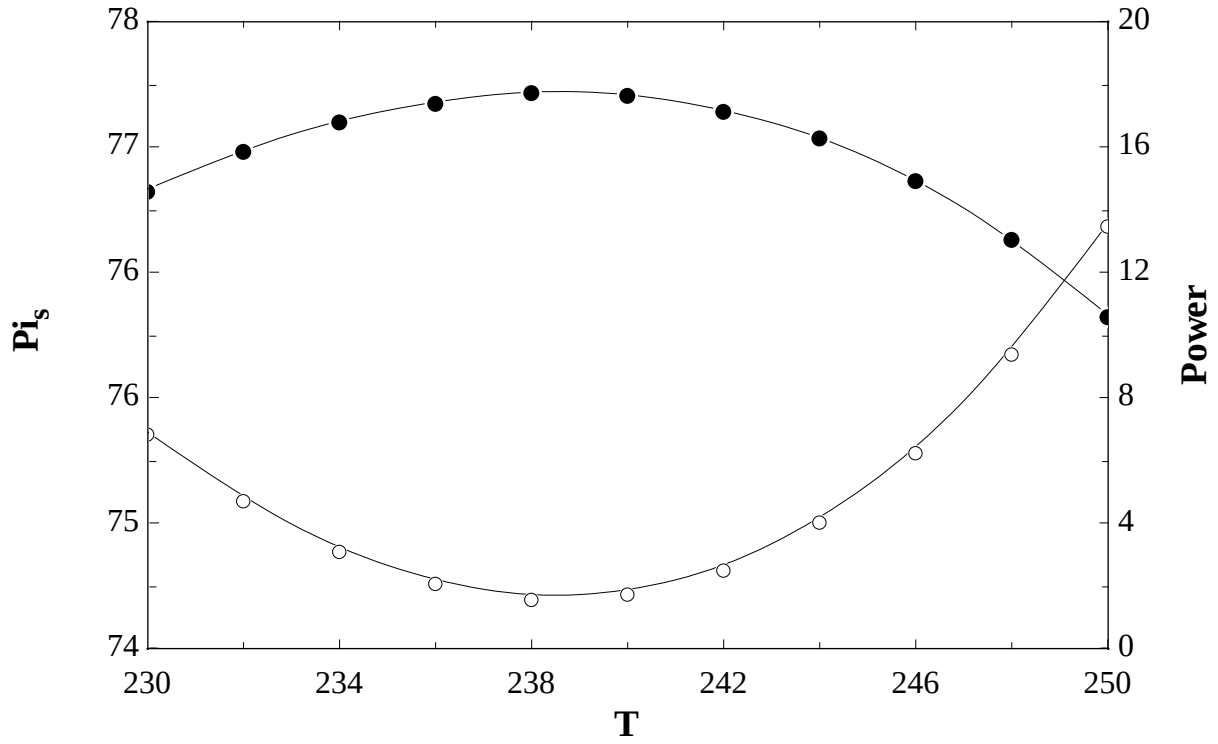
Total entropy production rate

$$Pi\_s = Pi\_sht + Pi\_sg + Pi\_satm + Pi\_sspace + Pi\_ssun$$

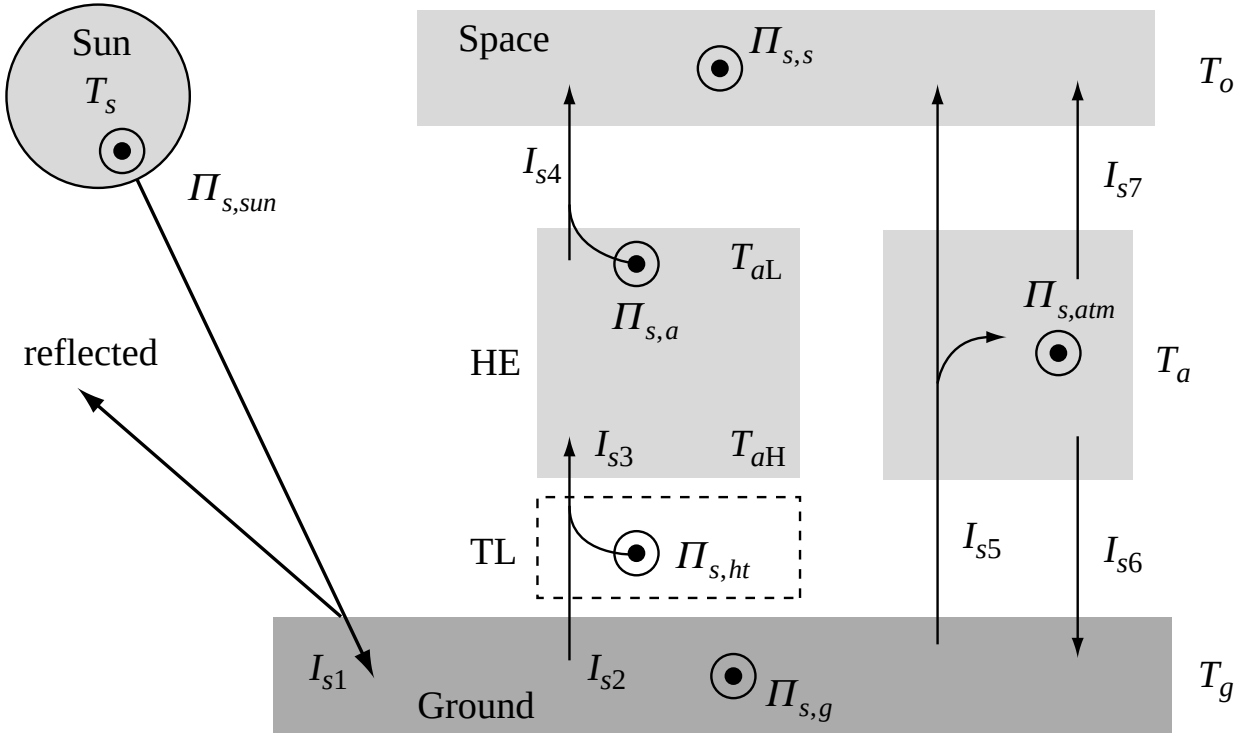
Power of winds

$$Power = (T\_aH - T\_aL) * I\_s3$$

SOLUTION



3 A MODEL OF THE GREENHOUSE EFFECT AND THE WIND ENGINE COMBINED



Model Using Second Law

Parameters	sigma = 5.67e-8
	a_ua = 0.25
	a_a = 0.87
	a_g = 1
	T_s = 5777
	T_o = 3
	R_s = 7.0e8
	d = 1.5e11
	albedo = 0.30
	h = 20
Constitutive laws for ground	
Rate of absorption	
of entropy from sun	A_sgs = 1/4*(1-albedo)*4/3*sigma*T_s^3*(R_s/d)^2
Emittance:	E_sg = 4/3*a_g*sigma*T_g^3
Radiosity:	B_sg = E_sg + (1-a_g)*B_sa
Flux to engine:	I_s2 = 1/T_g*h*(T_g-T_aH)
Entropy production rate:	Pi_sg = 1/4*E_sg + (3/4*T_a/T_g - 1)*a_g*B_sa + (3/4*T_s/T_g - 1)*A_sgs

Constitutive laws for atmosphere

Emittance:	E_sa = 4/3*a_a*sigma*T_a^3
Radiosity:	B_sa = E_sa
Entropy production rate:	Pi_sa = 2*1/4*E_sa + (3/4*T_g/T_a - 1)*a_a*B_sg

Constitutive laws for transfer layer

Entropy production rate:	Pi_sht = (T_g-T_aH)*I_s2/T_aH

Constitutive laws for sun

Entropy production rate:	Pi_ssun = 1/4*4/3*sigma*T_s^3*(R_s/d)^2

Constitutive laws for engine

Flux to space:	I_s4 = 4/3*a_ua*sigma*T_aL^3
Entropy production rate:	Pi_seng = 1/4*I_s4

Constitutive laws for space

Entropy production rate:	Pi_sspace = (3/4*T_aL/T_o-1)*I_s4 + (3/4*T_g/T_o-1)*(1-a_a)*E_sg + (3/4*T_a/T_o-1)*B_sa

Laws of balance

Ground:	A_sgs + a_g*B_sa + Pi_sg = E_sg + I_s2
Atmosphere:	a_a*B_sg + Pi_sa = 2*E_sa
Transfer layer:	I_s3 = I_s2 + Pi_sht
Engine:	I_s4 = I_s3 + Pi_seng

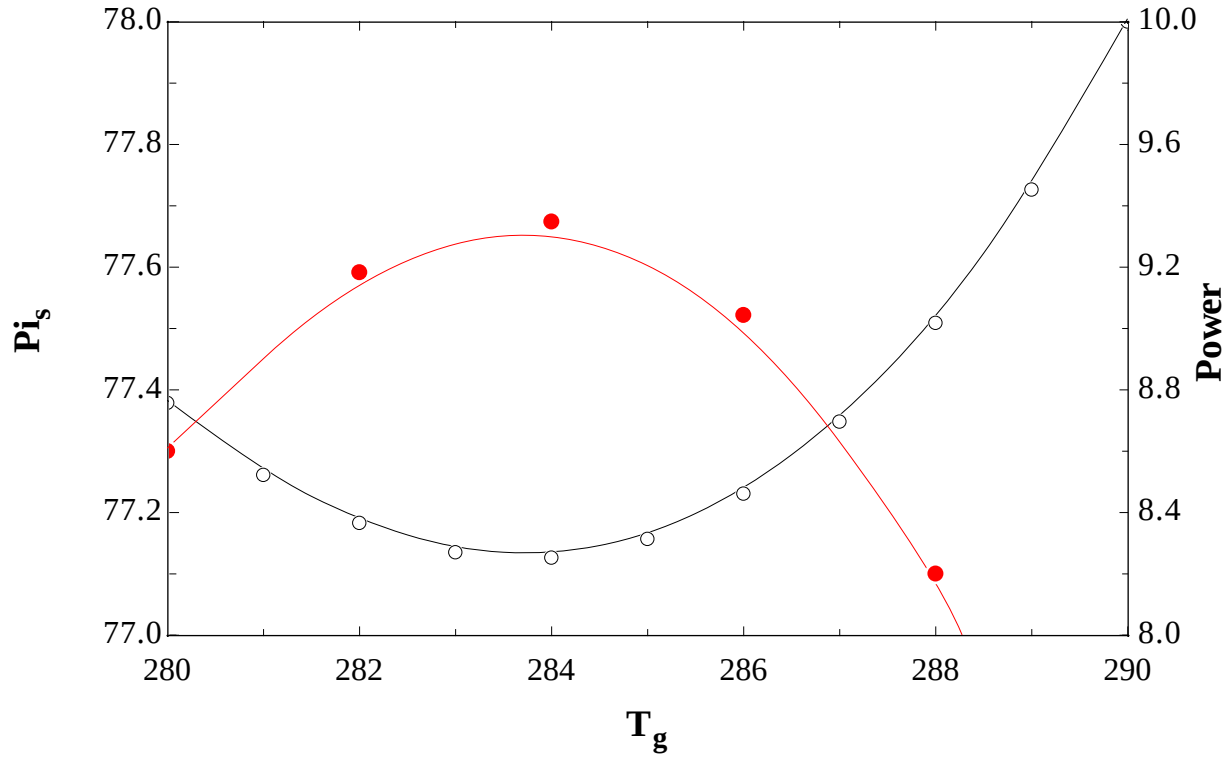
Total entropy production rate

$$Pi\_s = Pi\_sht + Pi\_sg + Pi\_seng + Pi\_sa + Pi\_sspace + Pi\_ssun$$

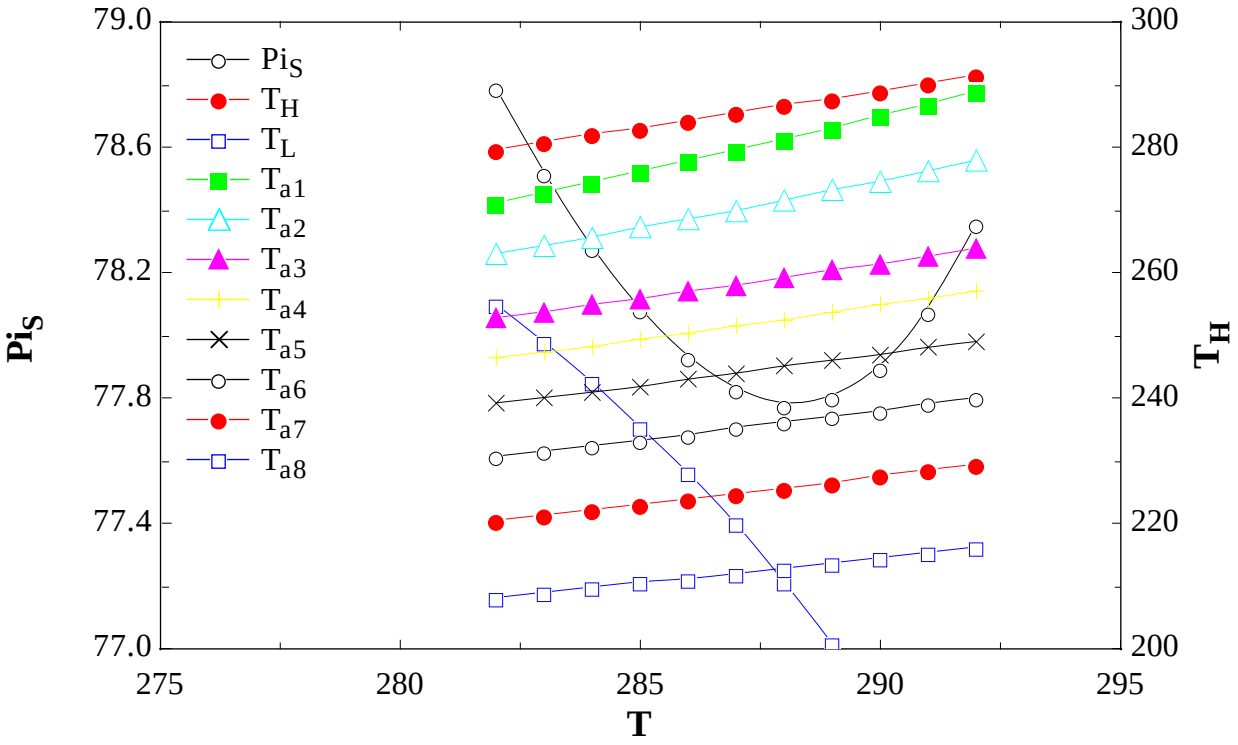
Power of winds

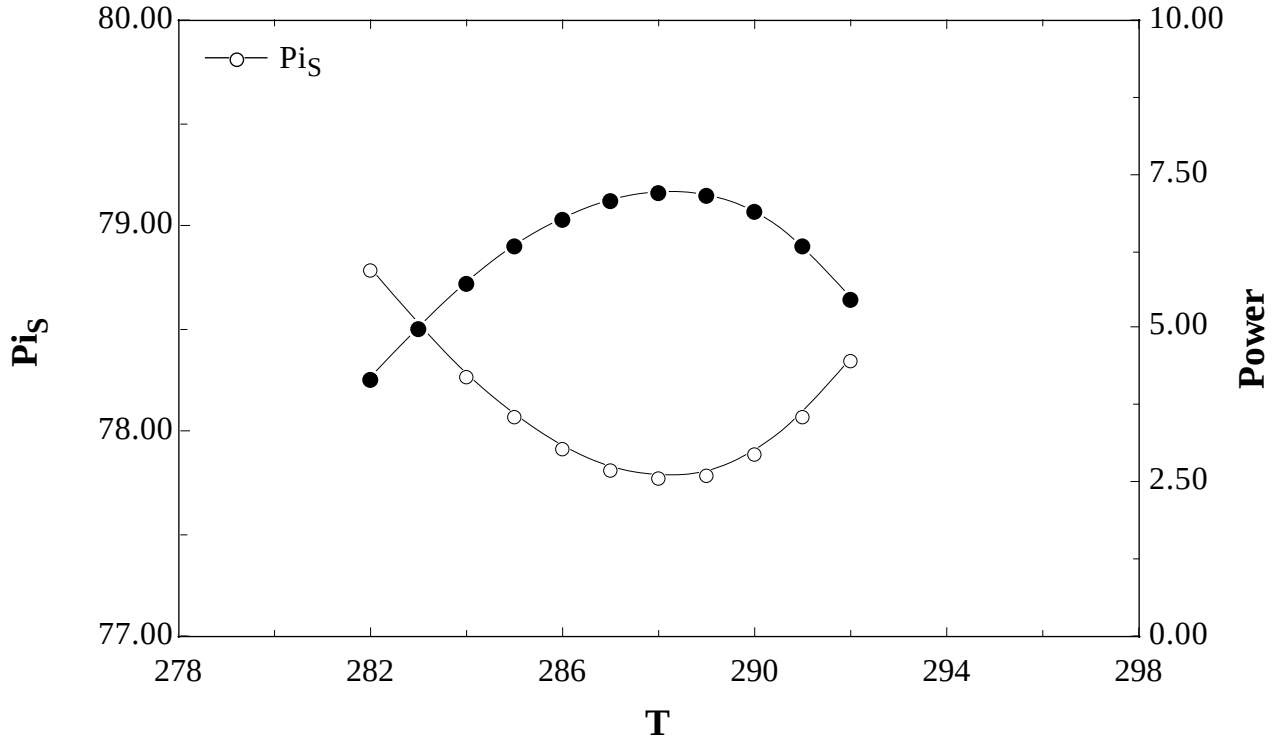
$$Power = (T\_aH - T\_aL) * I\_s3$$

SOLUTION



4 A LAYERED ATMOSPHERE

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79.00
78.00
77.00
10.00
7.50
5.00
2.50
0.00
278
282
286
290
294
298
T
Pi_S
Power
Pi_S

5 Background: The Dynamics of Heat